切变模量

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切变模量最终结果

【Latex代码】

钢丝直径d的平均值

$$  
\overline{d}=\frac{1}{n}\sum\_{i=1}^{n}d\_i=\frac{0.772+0.771+0.769+0.768+0.768+0.768+0.772+0.776+0.769+0.768}{10}\,\mathrm{mm}=0.7701\,\mathrm{mm}  
$$

钢丝直径d的标准差

$$  
\begin{aligned}  
\sigma\_{d}&=\sqrt{\frac{1}{n-1}\sum\_{i=1}^n\left(d\_i-\overline{d}\right)^2}\\  
&=\sqrt{\frac{(0.772-0.7701)^2+(0.771-0.7701)^2+(0.769-0.7701)^2+(0.768-0.7701)^2+(0.768-0.7701)^2+(0.768-0.7701)^2+(0.772-0.7701)^2+(0.776-0.7701)^2+(0.769-0.7701)^2+(0.768-0.7701)^2}{10-1}}\,\mathrm{mm}\\  
&=0.0026437\,\mathrm{mm}  
\end{aligned}  
$$

钢丝直径d的B类不确定度

$$  
\Delta\_{B,d}=\sqrt{\Delta\_\text{仪}^2+\Delta\_\text{估}^2}=\sqrt{0.01^2+0.005^2}\,\mathrm{mm}=0.01118\,\mathrm{mm}  
$$

钢丝直径d的展伸不确定度

$$  
\begin{aligned}  
U\_{d,P}&=\sqrt{\left(t\_P\frac{\sigma\_{d}}{\sqrt{n}}\right)^2+\left(k\_P\frac{\Delta\_{B,d}}{C}\right)^2}\\  
&=\sqrt{\left(2.26\times\frac{0.0026437}{\sqrt{10}}\right)^2+\left(1.96\times\frac{0.01118}{3}\right)^2}\,\mathrm{mm}\\  
&=7.5449 \times 10^{-3}\,\mathrm{mm},P=0.95  
\end{aligned}  
$$

环内直径d1的平均值

$$  
\overline{d1}=\frac{1}{n}\sum\_{i=1}^{n}d1\_i=\frac{79.52+79.72+79.52+79.76+79.74}{5}\,\mathrm{mm}=79.652\,\mathrm{mm}  
$$

环内直径d1的标准差

$$  
\begin{aligned}  
\sigma\_{d1}&=\sqrt{\frac{1}{n-1}\sum\_{i=1}^n\left(d1\_i-\overline{d1}\right)^2}\\  
&=\sqrt{\frac{(79.52-79.652)^2+(79.72-79.652)^2+(79.52-79.652)^2+(79.76-79.652)^2+(79.74-79.652)^2}{5-1}}\,\mathrm{mm}\\  
&=0.12133\,\mathrm{mm}  
\end{aligned}  
$$

环内直径d1的B类不确定度

$$  
\Delta\_{B,d1}=0.02\,\mathrm{mm}  
$$

环内直径d1的展伸不确定度

$$  
\begin{aligned}  
U\_{d1,P}&=\sqrt{\left(t\_P\frac{\sigma\_{d1}}{\sqrt{n}}\right)^2+\left(k\_P\frac{\Delta\_{B,d1}}{C}\right)^2}\\  
&=\sqrt{\left(2.78\times\frac{0.12133}{\sqrt{5}}\right)^2+\left(1.96\times\frac{0.02}{\sqrt{3}}\right)^2}\,\mathrm{mm}\\  
&=0.15253\,\mathrm{mm},P=0.95  
\end{aligned}  
$$

环外直径d2的平均值

$$  
\overline{d2}=\frac{1}{n}\sum\_{i=1}^{n}d2\_i=\frac{100+100+100+100+100}{5}\,\mathrm{mm}=100\,\mathrm{mm}  
$$

环外直径d2的标准差

$$  
\begin{aligned}  
\sigma\_{d2}&=\sqrt{\frac{1}{n-1}\sum\_{i=1}^n\left(d2\_i-\overline{d2}\right)^2}\\  
&=\sqrt{\frac{(100-100)^2+(100-100)^2+(100-100)^2+(100-100)^2+(100-100)^2}{5-1}}\,\mathrm{mm}\\  
&=0\,\mathrm{mm}  
\end{aligned}  
$$

环外直径d2的B类不确定度

$$  
\Delta\_{B,d2}=0.02\,\mathrm{mm}  
$$

环外直径d2的展伸不确定度

$$  
\begin{aligned}  
U\_{d2,P}&=\sqrt{\left(t\_P\frac{\sigma\_{d2}}{\sqrt{n}}\right)^2+\left(k\_P\frac{\Delta\_{B,d2}}{C}\right)^2}\\  
&=\sqrt{\left(2.78\times\frac{0}{\sqrt{5}}\right)^2+\left(1.96\times\frac{0.02}{\sqrt{3}}\right)^2}\,\mathrm{mm}\\  
&=0.022632\,\mathrm{mm},P=0.95  
\end{aligned}  
$$

钢丝长度L的平均值

$$  
\overline{L}=\frac{1}{n}\sum\_{i=1}^{n}L\_i=\frac{48.3+48.28+48.3+48.3+48.29}{5}\,\mathrm{cm}=48.294\,\mathrm{cm}  
$$

钢丝长度L的标准差

$$  
\begin{aligned}  
\sigma\_{L}&=\sqrt{\frac{1}{n-1}\sum\_{i=1}^n\left(L\_i-\overline{L}\right)^2}\\  
&=\sqrt{\frac{(48.3-48.294)^2+(48.28-48.294)^2+(48.3-48.294)^2+(48.3-48.294)^2+(48.29-48.294)^2}{5-1}}\,\mathrm{cm}\\  
&=0.0089443\,\mathrm{cm}  
\end{aligned}  
$$

钢丝长度L的B类不确定度

$$  
\Delta\_{B,L}=\sqrt{\Delta\_\text{仪}^2+\Delta\_\text{估}^2}=\sqrt{0.1^2+0.05^2}\,\mathrm{cm}=0.1118\,\mathrm{cm}  
$$

钢丝长度L的展伸不确定度

$$  
\begin{aligned}  
U\_{L,P}&=\sqrt{\left(t\_P\frac{\sigma\_{L}}{\sqrt{n}}\right)^2+\left(k\_P\frac{\Delta\_{B,L}}{C}\right)^2}\\  
&=\sqrt{\left(2.78\times\frac{0.0089443}{\sqrt{5}}\right)^2+\left(1.96\times\frac{0.1118}{3}\right)^2}\,\mathrm{cm}\\  
&=0.073887\,\mathrm{cm},P=0.95  
\end{aligned}  
$$

圆环质量m的平均值

$$  
\overline{m}=\frac{1}{n}\sum\_{i=1}^{n}m\_i=\frac{478.4+478.4+478.4+478.4+478.4}{5}\,\mathrm{g}=478.4\,\mathrm{g}  
$$

圆环质量m的标准差

$$  
\begin{aligned}  
\sigma\_{m}&=\sqrt{\frac{1}{n-1}\sum\_{i=1}^n\left(m\_i-\overline{m}\right)^2}\\  
&=\sqrt{\frac{(478.4-478.4)^2+(478.4-478.4)^2+(478.4-478.4)^2+(478.4-478.4)^2+(478.4-478.4)^2}{5-1}}\,\mathrm{g}\\  
&=0\,\mathrm{g}  
\end{aligned}  
$$

圆环质量m的B类不确定度

$$  
\Delta\_{B,m}=\sqrt{\Delta\_\text{仪}^2+\Delta\_\text{估}^2}=\sqrt{1^2+0.5^2}\,\mathrm{g}=1.118\,\mathrm{g}  
$$

圆环质量m的展伸不确定度

$$  
\begin{aligned}  
U\_{m,P}&=\sqrt{\left(t\_P\frac{\sigma\_{m}}{\sqrt{n}}\right)^2+\left(k\_P\frac{\Delta\_{B,m}}{C}\right)^2}\\  
&=\sqrt{\left(2.78\times\frac{0}{\sqrt{5}}\right)^2+\left(1.96\times\frac{1.118}{3}\right)^2}\,\mathrm{g}\\  
&=0.73045\,\mathrm{g},P=0.95  
\end{aligned}  
$$

周期T0的平均值

$$  
\overline{T0}=\frac{1}{n}\sum\_{i=1}^{n}T0\_i=\frac{2.5263+2.5309+2.5303}{3}\,\mathrm{s}=2.5291\,\mathrm{s}  
$$

周期T0的标准差

$$  
\begin{aligned}  
\sigma\_{T0}&=\sqrt{\frac{1}{n-1}\sum\_{i=1}^n\left(T0\_i-\overline{T0}\right)^2}\\  
&=\sqrt{\frac{(2.5263-2.5291)^2+(2.5309-2.5291)^2+(2.5303-2.5291)^2}{3-1}}\,\mathrm{s}\\  
&=0.0024908\,\mathrm{s}  
\end{aligned}  
$$

周期T0的B类不确定度

$$  
\Delta\_{B,T0}=\sqrt{\Delta\_\text{仪}^2+\Delta\_\text{估}^2}=\sqrt{0.0005^2+0.01^2}\,\mathrm{s}=0.010012\,\mathrm{s}  
$$

周期T0的展伸不确定度

$$  
\begin{aligned}  
U\_{T0,P}&=\sqrt{\left(t\_P\frac{\sigma\_{T0}}{\sqrt{n}}\right)^2+\left(k\_P\frac{\Delta\_{B,T0}}{C}\right)^2}\\  
&=\sqrt{\left(4.3\times\frac{0.0024908}{\sqrt{3}}\right)^2+\left(1.96\times\frac{0.010012}{3}\right)^2}\,\mathrm{s}\\  
&=9.0016 \times 10^{-3}\,\mathrm{s},P=0.95  
\end{aligned}  
$$

周期T1的平均值

$$  
\overline{T1}=\frac{1}{n}\sum\_{i=1}^{n}T1\_i=\frac{3.7488+3.7502+3.7486}{3}\,\mathrm{s}=3.7492\,\mathrm{s}  
$$

周期T1的标准差

$$  
\begin{aligned}  
\sigma\_{T1}&=\sqrt{\frac{1}{n-1}\sum\_{i=1}^n\left(T1\_i-\overline{T1}\right)^2}\\  
&=\sqrt{\frac{(3.7488-3.7492)^2+(3.7502-3.7492)^2+(3.7486-3.7492)^2}{3-1}}\,\mathrm{s}\\  
&=0.00087178\,\mathrm{s}  
\end{aligned}  
$$

周期T1的B类不确定度

$$  
\Delta\_{B,T1}=\sqrt{\Delta\_\text{仪}^2+\Delta\_\text{估}^2}=\sqrt{0.0005^2+0.01^2}\,\mathrm{s}=0.010012\,\mathrm{s}  
$$

周期T1的展伸不确定度

$$  
\begin{aligned}  
U\_{T1,P}&=\sqrt{\left(t\_P\frac{\sigma\_{T1}}{\sqrt{n}}\right)^2+\left(k\_P\frac{\Delta\_{B,T1}}{C}\right)^2}\\  
&=\sqrt{\left(4.3\times\frac{0.00087178}{\sqrt{3}}\right)^2+\left(1.96\times\frac{0.010012}{3}\right)^2}\,\mathrm{s}\\  
&=6.8902 \times 10^{-3}\,\mathrm{s},P=0.95  
\end{aligned}  
$$

扭转模量

$$  
D=\frac{\pi^{2} m \left(d\_{1}^{2} + d\_{2}^{2}\right)}{- 2 t\_{0}^{2} + 2 t\_{1}^{2}}=\frac{\pi^2\times 0.4784 \left(0.079652^2+0.1^2\right)}{-2\times 2.5291^2+2\times 3.7492^2}\,\mathrm{kg·m^2/s^2}=5.0374 \times 10^{-3}\,\mathrm{kg·m^2/s^2}  
$$

扭转模量D的延伸不确定度

$$  
\begin{aligned}  
U\_{D,P}&=\sqrt{\left(\frac{\partial D}{\partial m}U\_{m,P}\right)^2+\left(\frac{\partial D}{\partial d1}U\_{d1,P}\right)^2+\left(\frac{\partial D}{\partial d2}U\_{d2,P}\right)^2+\left(\frac{\partial D}{\partial t1}U\_{t1,P}\right)^2+\left(\frac{\partial D}{\partial t0}U\_{t0,P}\right)^2}\\  
&=\sqrt{\left(\frac{\pi^{2} \left(d\_{1}^{2} + d\_{2}^{2}\right)}{- 2 t\_{0}^{2} + 2 t\_{1}^{2}}U\_{m,P}\right)^2+\left(\frac{2 \pi^{2} d\_{1} m}{- 2 t\_{0}^{2} + 2 t\_{1}^{2}}U\_{d1,P}\right)^2+\left(\frac{2 \pi^{2} d\_{2} m}{- 2 t\_{0}^{2} + 2 t\_{1}^{2}}U\_{d2,P}\right)^2+\left(- \frac{4 \pi^{2} m t\_{1} \left(d\_{1}^{2} + d\_{2}^{2}\right)}{\left(- 2 t\_{0}^{2} + 2 t\_{1}^{2}\right)^{2}}U\_{t1,P}\right)^2+\left(\frac{4 \pi^{2} m t\_{0} \left(d\_{1}^{2} + d\_{2}^{2}\right)}{\left(- 2 t\_{0}^{2} + 2 t\_{1}^{2}\right)^{2}}U\_{t0,P}\right)^2}\\  
&=\sqrt{\left(\frac{\pi^2\times \left(0.079652^2+0.1^2\right)}{-2\times 2.5291^2+2\times 3.7492^2}\times 0.00073045\right)^2+\left(\frac{2\times \pi^2\times 0.079652\times 0.4784}{-2\times 2.5291^2+2\times 3.7492^2}\times 0.00015253\right)^2+\left(\frac{2\times \pi^2\times 0.1\times 0.4784}{-2\times 2.5291^2+2\times 3.7492^2}\times 2.2632e-05\right)^2+\left(-\frac{4\times \pi^2\times 0.4784\times 3.7492 \left(0.079652^2+0.1^2\right)}{\left(-2\times 2.5291^2+2\times 3.7492^2\right)^2}\times 0.0068902\right)^2+\left(\frac{4\times \pi^2\times 0.4784\times 2.5291 \left(0.079652^2+0.1^2\right)}{\left(-2\times 2.5291^2+2^2\right)^2}\right)^2}\,\mathrm{kg·m^2/s^2}\\  
&=4.6564 \times 10^{-5}\,\mathrm{kg·m^2/s^2},P=0.95  
\end{aligned}  
$$

扭转模量最终结果

$$  
D=\left(0.00504 \pm 0.00005\right)\,\mathrm{kg·m^2/s^2}  
$$

切变模量

$$  
G=\frac{16 \pi L m \left(d\_{1}^{2} + d\_{2}^{2}\right)}{d^{4} \left(- t\_{0}^{2} + t\_{1}^{2}\right)}=\frac{16 \pi\times 0.48294\times 0.4784 \left(0.079652^2+0.1^2\right)}{0.0007701^4\times \left(-2.5291^2+3.7492^2\right)}\,\mathrm{kg/(m·s^2)}=7.0455 \times 10^{10}\,\mathrm{kg/(m·s^2)}  
$$

切变模量G的延伸不确定度

$$  
\begin{aligned}  
U\_{G,P}&=\sqrt{\left(\frac{\partial G}{\partial L}U\_{L,P}\right)^2+\left(\frac{\partial G}{\partial m}U\_{m,P}\right)^2+\left(\frac{\partial G}{\partial d1}U\_{d1,P}\right)^2+\left(\frac{\partial G}{\partial d2}U\_{d2,P}\right)^2+\left(\frac{\partial G}{\partial d}U\_{d,P}\right)^2+\left(\frac{\partial G}{\partial t1}U\_{t1,P}\right)^2+\left(\frac{\partial G}{\partial t0}U\_{t0,P}\right)^2}\\  
&=\sqrt{\left(\frac{16 \pi m \left(d\_{1}^{2} + d\_{2}^{2}\right)}{d^{4} \left(- t\_{0}^{2} + t\_{1}^{2}\right)}U\_{L,P}\right)^2+\left(\frac{16 \pi L \left(d\_{1}^{2} + d\_{2}^{2}\right)}{d^{4} \left(- t\_{0}^{2} + t\_{1}^{2}\right)}U\_{m,P}\right)^2+\left(\frac{32 \pi L d\_{1} m}{d^{4} \left(- t\_{0}^{2} + t\_{1}^{2}\right)}U\_{d1,P}\right)^2+\left(\frac{32 \pi L d\_{2} m}{d^{4} \left(- t\_{0}^{2} + t\_{1}^{2}\right)}U\_{d2,P}\right)^2+\left(- \frac{64 \pi L m \left(d\_{1}^{2} + d\_{2}^{2}\right)}{d^{5} \left(- t\_{0}^{2} + t\_{1}^{2}\right)}U\_{d,P}\right)^2+\left(- \frac{32 \pi L m t\_{1} \left(d\_{1}^{2} + d\_{2}^{2}\right)}{d^{4} \left(- t\_{0}^{2} + t\_{1}^{2}\right)^{2}}U\_{t1,P}\right)^2+\left(\frac{32 \pi L m t\_{0} \left(d\_{1}^{2} + d\_{2}^{2}\right)}{d^{4} \left(- t\_{0}^{2} + t\_{1}^{2}\right)^{2}}U\_{t0,P}\right)^2}\\  
&=2.8389 \times 10^{9}\,\mathrm{kg/(m·s^2)},P=0.95  
\end{aligned}  
$$

切变模量最终结果

$$  
G=\left(7.05 \pm 0.28\right) \times 10^{10}\,\mathrm{kg/(m·s^2)}  
$$